CHAPTRE SEVEN

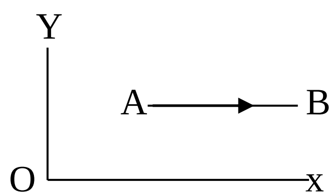
VECTORS

- A vector is a physical quantity which has both magnitude and direction.
- Example are
  a. A force of 20N acting North.
  b. A velocity of 5km/h East.

Types of vectors:

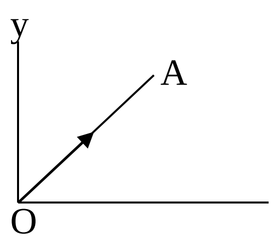
- In general the are two types and these are
  i. Free vector.
  ii. Position vector.

Free vector:



- A free vector is a vector which does not pass through any specific position.
- They are usually represented by small letters e.g $e.g\ \underset{\sim}{a}, \underset{\sim}{b}\ \ and\ \underset{\sim}{c}$

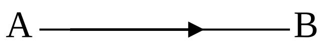
Position vector :



This is a vector which passes through the origin or a specified point.

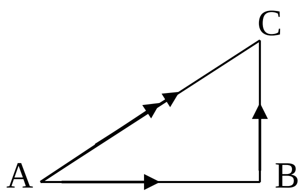
Vector notation:

- A vector may be represented by a line segment as shown next:



- This given vector can be represented by $\overrightarrow{AB}, \overline{AB}, \underline{AB}, \widetilde{AB}, \underset{\sim}{AB}$.

The Triangle law:



According to the triangle law, $\overline{AC} = \overline{AB} + \overline{BC} \Longrightarrow \overline{AB} = \overline{AC} - \overline{BC}\ and\ \overline{BC} = \overline{AC} - \overline{AB}$

The unit vector:

- This is a vector whose magnitude is one in the direction under consideration.
- The unit vector along a vector $\vec{a}$is written as $\hat{a}$
- Also the unit vector along a vector $\overrightarrow{b}$ is written as $\hat{b}$
- The unit vector along the vector $\overline{BC}$ is written as $\widehat{BC}$
- Consider the vector A⟶B = 1
- The vector is written as $\overrightarrow{AB}$and its unit vector is written as $\widehat{AB}$.

Equal vectors:

- Two vectors are said to be equal if their magnitudes and directions are equal
- Example are $\overline{AB} = 50km/hE\ and\ \overline{CD} = 50km/h\ E$.

The negative vector:

- The negative of the vector $\underset{\sim}{a}\ is$ written as –a
- If $^{-}\underset{\sim}{a}\ is$ the negative vector of the vector $\underset{\sim}{a},\ then\ \underset{\sim}{a}+(^{-}\underset{\sim}{a}) = \underset{\sim}{o}$.
- The vector $^{-}\underset{\sim}{a}\ is\ a\ vector$ of the same magnitude as $\underset{\sim}{a}$, but it is opposite in direction.
- It must be noted that $\overline{AB} + \overline{BA} = \underset{\sim}{o}$.

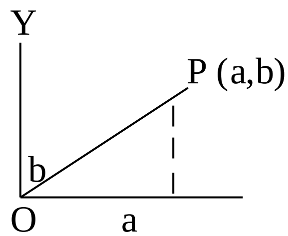
- Also if $\underset{\sim}{b} = \overrightarrow{CD}$, *then* $-\underset{\sim}{b} = \overrightarrow{DC}$, *and* $\overline{CD} + \overline{DC} = \underset{\sim}{0}$.
- If we consider a vector $\overline{CD}$, then its negative vector is $\overline{DC}$.

The zero vector (null vector):

- This is a vector where magnitude is zero and its direction is undefined.
- It is represented by $\underset{\rightarrow}{0} = \binom{0}{0}$

Notation of the magnitude of a vectors:

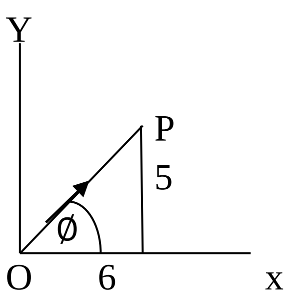
- If $\overline{AB}$ is a vector, then its magnitude is written as $|\overrightarrow{AB}|$
- Similarly the magnitude of the vector $\vec{b}$ is written as $|\vec{b}|$
- If $\overline{OP} = \binom{a}{b}$, *then its* magnitude $= |\overline{OP}| = \sqrt{a^2 + b^2}$



Q1. i. If OP = $\binom{6}{5}$, *find the* magnitude of $\overline{OP}$.

ii. Find ∅ the angle between $\overline{OP}$ and the x – axis

Soln.



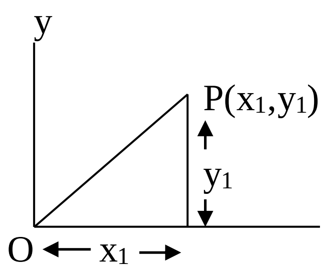
i. $|\overrightarrow{OP}| = \sqrt{6^2 + 5^2} = \sqrt{61} = 7.8$

ii. $\tan \emptyset = {}^{5}/_{6} \Rightarrow \tan \emptyset = 0.83 \Rightarrow \emptyset = tan^{-1} 0.83 \Rightarrow \emptyset = 40.$

Scalar multiplication of vector:

- If ^ is the scalar and $\bar{a}$ is the vector, then the scalar x the vector $= \text{^}\vec{a}$
- When a scalar multiplies a vector, the product is also a vector, and for this reason $\text{^}\bar{a}$ is also a vector.
- The vector $\text{^}\underset{\sim}{a}$ is parallel to $\underset{\sim}{a}$, and is in the same direction as $\underset{\sim}{a}$,but has ^ times the magnitude of $\underset{\sim}{a}$.
- For example the vectors $\vec{a}\ and\ 2\vec{a}$ have the same direction.
  i.e $|\vec{a}|$ $|2\vec{a}|$
- But the vectors $\vec{a}$ and and $-2\vec{a}$ are opposite in direction.
- $\text{^}(\vec{a}+\vec{b}) = \text{^}\vec{a} + \text{^}\vec{b}$, e.g $6(\underset{\sim}{a}+\underset{\sim}{b}) = 6\underset{\sim}{a}+6\underset{\sim}{b}$
- Also (2 + 4) $\vec{a} = 2\vec{a} + 4\vec{a}$
- Finally $\text{^}_1(\text{^}_2\vec{a}) = \text{^}_1\text{^}_2\vec{a}$, e.g $3(2\vec{a}) = 6\vec{a}$

N/B:



- If $P(x_1,y_1)$ is a point in the x – y plane, then the position vector of P relative to the origin, O is defined by $\overrightarrow{OP} = \binom{x_1}{y_1}$
- Also if A = (0,6), then $\overrightarrow{OA} = \binom{0}{6}$

Q2. Find the numbers m and n such that

M$\binom{3}{5} + n\binom{2}{1} = \binom{4}{9}$

Soln.

$$M\begin{pmatrix}3\\5\end{pmatrix} + n\begin{pmatrix}2\\1\end{pmatrix} = \begin{pmatrix}4\\9\end{pmatrix} \Rightarrow \begin{pmatrix}3m\\5m\end{pmatrix} + \begin{pmatrix}2n\\n\end{pmatrix} = \begin{matrix}4\\9\end{matrix}$$

$\Rightarrow 3m + 2n = 4 \ldots\ldots eqn(1).$

$5m + n = 9 \ldots\ldots.\ldots eqn(2)$

Solve eqns (1) and (2) simultaneously

$\Longrightarrow m = 2 \; and \; n = -1$

Q3. If mp + nq = $\binom{4}{3}$, $find \; m \; and \; n$ where m and n are scalar, given that p = $\binom{2}{3} \; and \; q = \binom{2}{5}$

Soln.

p = $\binom{2}{3} \; and \; q = \binom{2}{5} \; but \; mp + nq = \binom{4}{3}$

$\Longrightarrow m\begin{pmatrix}2\\3\end{pmatrix} + n\begin{pmatrix}2\\5\end{pmatrix} = \begin{pmatrix}4\\3\end{pmatrix} \Longrightarrow \begin{pmatrix}2m\\3m\end{pmatrix} + \begin{pmatrix}2n\\5n\end{pmatrix} = \begin{pmatrix}4\\3\end{pmatrix}$

$\Longrightarrow 2m + 2n = 4 - (1)$

$3m + 5n = 3 - (3)$

Solve eqns (1) and (2) simultaneously to get the values of m and n.

Q4. If r = $\binom{3}{1} and \; s = \binom{-2}{1}$, evaluate 6(r + 25)

Soln.

Consider 6(r + 2s), solve what is inside the bracket first $\Longrightarrow r + 2s = \binom{3}{1} + \binom{-2}{1} = \binom{3}{1} + 2\binom{-4}{2} \Longrightarrow r + 2s = \binom{3+\overline{4}}{1+2} = \binom{-1}{3} \Longrightarrow 6(r + 2s) = 6\binom{-1}{3} = \binom{-6}{18}$

Q5. If p = $\binom{1}{2}, q = \binom{-2}{3} \; and \; r = \binom{1}{1}$, find 2p – q + r

Soln.

2p – q + r = $2\binom{1}{2} - \binom{-2}{3} + \binom{1}{1} = \binom{2}{4} - \binom{-2}{3} + \binom{1}{1} = \binom{2+2+1}{4-3+1} = \binom{5}{2} \Longrightarrow 2p - q + r = \binom{5}{2}$.

Q6. If the vector p = $\binom{2}{3}, q = \binom{2}{5} and \; r = \frac{1}{2}(q - p)$,

Find the vector r.

Soln.

r = $\frac{1}{2}(q-p) \Longrightarrow r = \frac{1}{2}\left\{\binom{2}{5} - \binom{2}{3}\right\} \Longrightarrow r = \frac{1}{2}\begin{Bmatrix} 2-2 \\ 5-3 \end{Bmatrix} = \frac{1}{2}\begin{Bmatrix} 0 \\ 2 \end{Bmatrix} = \begin{Bmatrix} \frac{1}{2}(0) \\ \frac{1}{2}(2) \end{Bmatrix} = \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \Longrightarrow r = \begin{Bmatrix} 0 \\ 1 \end{Bmatrix}$

N/B: Given the points A and B, then $\overrightarrow{AB} = B - A$.

Examples: If A = $\binom{5}{2}$ $and\ B = \binom{10}{6}, then\ \overrightarrow{AB} = B - A = \binom{10}{6} - \binom{5}{2} = \binom{10-5}{6-2} = \binom{5}{4}$

Also if C = $\binom{4}{2}$ $and\ D = \binom{6}{1}, then\ \overrightarrow{CD} = D - C = \binom{6}{1} - \binom{4}{2} = \binom{6-4}{1-2} = \binom{2}{-1} \Longrightarrow \overrightarrow{CD} = \binom{2}{-1}$

Q7. If A = (4, 5) and B = (6, 2), find $\overline{AB}$

Soln.

A = (4,5) $\Longrightarrow A = \binom{4}{5}$. Also B = (6, 2) $\Longrightarrow B = \binom{6}{2}. \overline{AB} = B - A = \binom{6}{2} - \binom{4}{5} = \binom{6-4}{2-5} = \binom{2}{-3} \Longrightarrow \overline{AB} = \binom{2}{-3}$.

N/B: If $\overline{AB} = \binom{4}{2} \Longrightarrow \overline{BA} = -\overline{AB} = -\binom{4}{2} = \binom{-4}{-2}$

Also if $\overrightarrow{CD} = \binom{-2}{5} \Longrightarrow \overrightarrow{DC} = -\overrightarrow{CD} = -\binom{-2}{5} = \binom{2}{-5}$

Q8. If A and B are the points (2, 1) and (1, 2) respectively, find $\overline{AB}\ and\ \overline{BA}$

Soln.

A = $\binom{2}{1}$ $and\ B = \binom{1}{2} \Longrightarrow \overrightarrow{AB} = B - A = \binom{1}{2} - \binom{2}{1} = \binom{1-2}{2-1} = \binom{-1}{1}$

$\overline{BA} = -\overline{AB} = -\binom{-1}{1} = \binom{1}{-1}$.